"Definitions," says the Report, "should not be based upon crude images, affording little upon which reasoning may lay hold. For example, 'An angle is the opening between two lines which meet.' Instead, define an angle as the figure formed by two rays having a common origin." A demonstration in which we use information obtained by looking at a figure is not of the highest order.

The one serious slip in the Report is the sentence: "Also, in 'A line perpendicular to each of two intersecting lines (at their intersection) is perpendicular to their plane,' we assume that two intersecting lines have a common perpendicular though we cannot justify the assumption by any previous proposition." Halsted's Rational Geometry here makes no assumption whatever. Its figure for this proposition is already covered by the preceding proposition: On any straight to put two planes; and the problem: To erect a perpendicular to a straight from any point on it.

We must agree with the Report, that the treatment of mensuration in most texts is extremely unfortunate. In fact measurement in terms of a common unit at once introduces incommensurability and irrational numbers. No geometry exists in which irrational numbers are adequately treated. Halsted's Rational Geometry outwits the difficulty.

The committee recommends that a critical course in elementary geometry be offered in courses of study in colleges.

DEPARTMENTS.

SOLUTIONS OF PROBLEMS.

ALGEBRA.

Remarks on Two Solutions of Problem 89. By G. A. MILLER.

The following solutions present very instructive examples of fallacious reasoning and arriving at the answer by a remarkable coincidence. As the problem is so well known, and these solutions are said to have appeared in other scientific journals it seems desirable to enter into some details. The problem is as follows:

Solve by quadratics, $x^2+y=7\ldots(1)$, $x+y^2=11\ldots(2)$.

We shall first speak of the solution given on page 37, Volume VI, of this journal. To make the matter as clear as possible we shall employ the language of analytic geometry. The problem is to find the common points (or at least one of them) of two intersecting parabolas. The author of the solution in question subtracts (2) from (1), and thus obtains the equation of an equilateral hyperbola containing the four common points of the given parabolas. The equation of this hyperbola is

$$y^2-x^2-(y-x)=4\ldots(3).$$

Substituting a for $x+y$ and b for $y-x$ tnis equation reduces to

$$ab-b=4\ldots(4).$$

While hyperbola (4) is satisfied by an infinite number of pairs of values for a and b, the author arrives at the pair $a=5$, $b=1$ by taking the following steps.

From (4) we may obtain by transposing and squaring,

$$a^2b^2=16+8b+b^2\ldots(5),$$

and also

$$-10ab+25=-15-10b\ldots(6).$$

Adding (5) and (6) there results

$$(ab-5)^2=(1-b)^2,$$

or

$$ab-5=1-b,\ ab+b=6\ldots(7).$$

Combining (4) and (7), it follows that $b=1$ and $a=5$.

The author has thus found one point on hyperbola (4), and the remarkable coincidence is that this is a point of intersection of the parabolas (1) and (2). Hence $x+y=5$, $y-x=1$ lead to $x=2$, $y=3$, which is a solution of the equations. That the method is erroneous follows directly from the fact that the same auxiliary equations could be obtained from

$$x^2+y=k\ldots(1),$$
$$x+y^2=k+4\ldots(2),$$

where k is arbitrary. If we make $k=0$, for instance, it is evident that $x=2$, $y=3$ does not satisfy the system. The solution is a good illustration of blindly manipulating algebraic expressions. Equations (3) and (4) represent a hyperbola which goes through the points whose co-ordinates are desired, and the rest of the solution is merely a very laborious method of finding the co-ordinates of one point on this hyperbola. As the number of points on the hyperbola is infinite, while only four of these points are common to the given parabolas, the probability that this method should lead to a correct result in a given problem is zero, and yet it happened to do so in the present instance.

The second solution of the same problem, to which we desire to call attention, is published on page 13 of the same volume. Equations (1) and (2) are written as follows:

$$x^2-9=2-y=d, \text{ by assumption,}$$
$$x-3=4-y^2=sd, \text{ by assumption.}$$

Hence

$$x^2-9=(x-3)/s=x/s-3/s.$$

Completing the square we have

$$x^2-x/3+1/4s^2=9-3/s+1/4s^2.$$

Hence $$x-1/2s=3-1/2s, \text{ or } x=3.$$

The fallacy will at once appear if it is observed that the same arguments could be used with respect to the two equations

$$x^2+y=9+\alpha\ldots(1),$$
$$y^2+x=3+\beta\ldots(2).$$

These may be written as follows:

$$x^2-9=\alpha-y=d\ldots(3),$$
$$x-3=\beta-y^2=sd\ldots(4).$$

As x cannot be equal to 3 for an arbitrary pair of values of α, β it follows that the method is fallacious. Just as in the preceding solution it is implicitly assumed that any point on $x^2-9=(x-2)/s$ must be common to the given parabolas. And the probability that this method should lead to the correct result in a given problem is again zero.

278. Proposed by W. J. GREENSTREET, M. A., Editor of The Mathematical Gazette, Stroud, England.

$xyz(\Sigma x)^2<3\Sigma y^2z\,\Sigma yz^2$, if x, y, z are positive.

Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.

$$\frac{x^2y+y^2z+z^2x}{3}>(x^2y.y^2z.z^2x)^{1/3}>xyz. \quad \text{Also } \frac{xy^2+yz^2+zx^2}{3}>xyz.$$

$\therefore 3\Sigma y^2z\Sigma yz^2>27x^2y^2z^2>xyz.27xyz.$
$(x+y+z)^2$ is the greatest when $x=y=z$, and is then equal to $9x^2$.
$\therefore (x+y+z)^2<27xyz. \quad \therefore xyz[\Sigma(x)]^2<3\Sigma y^2z\Sigma yz^2.$

279. Proposed by THEODORE L. DE LAND, Treasury Department, Washington, D. C.

The United States Panama Canal Bonds were issued, to date August 1, 1906, and will mature on August 1, 1936; and they bear interest at the rate of 2% per annum, payable quarterly, on the first day of November, 1906, and the first day of February, May, and August, 1907, and so on for each succeeding quarter, until the bonds mature, when the principal will be paid at par with the last quarter's interest. The coupon bonds of this loan were quoted on the New York Stock Exchange, at 10.30 a. m., on December 17, 1906, at $103\frac{3}{4}$ bid and $104\frac{3}{4}$ asked.

Required: The rate of interest per annum, payable quarterly, an investor would *realize* if he purchased the Panama bonds on December 17, 1906, and could reinvest his interest income, quarterly, at the *realized* rate.